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Naturally Degenerate Right Handed Neutrinos

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Abstract

In the context of supersymmetric theories, a weakly broken gauged SO(3) flavour symmetry is used to produce two highly degenerate right handed (RH) neutrinos. It is then shown that this SO(3) flavour symmetry is compatible with all fermion masses and mixings if it is supplemented with a further SU(3) flavour symmetry. A specific Susy breaking model is used to generate the light neutrino masses as well as a natural model of TeV scale resonant leptogenesis.

1 Introduction

TeV scale leptogenesis is an important alternative to the leptogenesis model associated with the seesaw mechanism [1]. The standard see-saw mechanism [2] prescribes heavy RH neutrinos and it is the decay of these states that can lead to an asymmetry in lepton number. At this high scale the Hubble constant, H, is generally larger than the decay widths of the RH neutrino states and consequently they decay out of thermal equilibrium. This departure from thermal equilibrium ensures that any asymmetry produced is not immediately washed out by inverse decays or any scatterings that involve the RH neutrino. However, due to the high mass scale of the RH neutrinos the see-saw mechanism and its' associated leptogenesis mechanism are difficult to directly test. This is in contrast to TeV scale theories of neutrino mass generation and leptogenesis [3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. One of the more attractive features of low scale theories is the possibility of being able to directly test components of the model.

A TeV scale theory will have a small Hubble constant. We require that the various scatterings which can suppress an asymmetry be under control. At these low scales gauge scatterings are very fast, consequently a singlet of all low energy gauge symmetries is preferred for the decaying particle. Considering standard thermal leptogenesis, one can think about various possibilities with decaying singlet particles at low scales: a large degeneracy of masses between the decaying particles [3, 4, 5, 6, 7, 8, 9, 12]; a hierarchy between the couplings of real and virtual particles in the one loop leptogenesis diagrams [6, 10]; or three body decays of the heavy particles with suppressed two body decays [6] (for related work in leptogenesis see, [13, 14]).

In this letter we will concentrate on the possibility of decaying TeV scale RH neutrinos with a large degeneracy in their masses. This framework suffers from various significant difficulties:

1) Seesaw type neutrino masses require tiny couplings and consequently will usually induce a tiny CP asymmetry.

2) We need the decay width of the particle which generates the asymmetry to be less than H, so that the particle decay will be out of thermal equilibrium and any asymmetry produced is not immediately washed out. This again requires tiny Yukawa couplings of order $10^{-6} - 10^{-7}$. Such small couplings need justification.

3) In a generic seesaw model there is no explanation why the RH neutrinos would have such a small mass ($M_N \sim$ TeV).

4) In order to compensate the large suppression of the asymmetry induced by these tiny couplings, an extremely tiny mass splitting is required between two RH neutrino masses giving a resonant behaviour in the RH neutrino propagator. The degree of degeneracy required has to be of order $(M_{N_1} - M_{N_2})/(M_{N_1} + M_{N_2}) < 10^{-10}$ [7]. This level of degeneracy needs to be physically motivated.

5) Finally, as a result of the constraints 1) and 2) the tiny Yukawa couplings imply that the RH neutrino production cross sections are very small. Which means that even at low scales the theory may not be testable.

In this letter we will argue, extending the arguments of [9, 15, 16], that these potential difficulties can be overcome. In the context of broken susy Ref. [9] considered two or more quasi-degenerate RH neutrinos. In this case the asymmetry can be significantly enhanced through a resonant behaviour of the propagator of the virtual particle in the leptogenesis self-energy diagram [9]. This model possesses a natural explanation for both tiny Yukawa couplings and TeV scale RH neutrinos (see Ref [16] for more details). Now one would like to form a natural explanation for the high degree of degeneracy in the RH neutrino spectrum.

In the following section we propose an SO(3) flavour symmetry which can be used to produce two exactly degenerate RH neutrinos [1]. In Section 3 a toy model is outlined where the SO(3) flavour symmetry is embedded into the susy breaking model described in Refs.[9] and [16]. Utilising a further SU(3) flavour symmetry it is shown that all fermionic standard model sectors including neutrino masses and mixings are compatible with this SO(3) flavour symmetry[2]. Following this we go on to describe a natural and successful model of TeV scale resonant leptogenesis. Our conclusions are contained in Section 4, while two appendices contain technical details of the models presented.

2 The SO(3) flavour symmetry

We assume the minimal supersymmetric standard model (MSSM) with the addition of standard model singlet RH neutrino chiral supermultiplets, N_i. Under a gauged SO(3) flavour symmetry N_i transforms as a triplet, where i=1,2,3 (and all other Roman indices) are SO(3) labels. All other MSSM chiral supermultiplets transform as singlets under this SO(3) flavour symmetry.

We need to spontaneously break the SO(3) flavour symmetry [3]. This is performed by two flavon fields, ζ and ξ, developing vacuum expectation values (VEVs). Each field is a triplet under the SO(3) flavour symmetry but a singlet under the standard model gauge group.

2.1 Degenerate Right Handed Neutrinos

RH neutrino masses can be generated via the superpotential or the Kahler potential depending on how exactly the scale of their masses is realised. This letter concentrates on the generation of TeV scale RH neutrinos via non-renormalisable operators arising from the Kahler potential. However, as a simple example of how the SO(3) flavour symmetry can generate degenerate RH neutrinos it is appropriate to study the mechanism in the context of an effective superpotential. Using the flavon field discussed above we can write

[1] An SO(3) symmetry has been previously used in connection with quasi-degenerate light neutrinos, see Ref.[17].

[2] In this paper we want to argue that there exists a model with naturally degenerate RH neutrinos justified by a symmetry, which is compatible with the Standard Model. It is not claimed that this is the most minimal solution.

[3] Using a gauged SO(3) symmetry means that any potentially dangerous massive vectors are avoided.

down,

$$M_N \int d^2\theta \left(h_1 N_i N_i + \frac{1}{M_f^2} h_2 N_i \zeta_i N_j \xi_j + \frac{1}{M_f^4} h_3 \epsilon_{ijk} N_i \zeta_j \xi_k \epsilon_{lmn} N_l \zeta_m \xi_n \right) \tag{1}$$

where ϵ_{ijk} is the usual antisymmetric tensor, M_N is the RH neutrino scale, M_f is the cut off scale, which we assume is the mass scale of some heavy fields that have been integrated out, all hs are undetermined $O(1)$ parameters and we assume the R-parities of ζ and ξ are equal in magnitude but opposite in sign.

We assume the two flavon fields develop VEV structures given as,

$$\langle\zeta\rangle = \begin{pmatrix} A \\ iA \\ 0 \end{pmatrix} \quad , \qquad \langle\xi\rangle = \begin{pmatrix} D \\ -iD \\ 0 \end{pmatrix} \tag{2}$$

where A and D are related and can be complex. The alignment of these two VEVs is crucial for the generation of degenerate RH neutrinos and is presented in the next section. It is assumed that the VEVs of ζ and ξ are comparable to the high scale so that $a \equiv A/M_f$ and $d \equiv D/M_f$ are not much less than one.

Allowing the two fields to acquire their VEVs the RH neutrino mass matrix has the form,

$$M_N^{sp} \sim \begin{pmatrix} h_1 + h_2 ad & 0 & 0 \\ 0 & h_1 + h_2 ad & 0 \\ 0 & 0 & h_1 + h_3 4a^2 d^2 \end{pmatrix} \tag{3}$$

where a minus sign has been absorbed into the definition of h_3. There are further terms that can be written down in addition to those in equation (1) but none of these give either non-diagonal or differing (1,1), (2,2) entries in the mass matrix. Consequently we produce two exactly degenerate RH neutrinos [4].

2.2 Vacuum Alignment

The crucial part of this model is the vacuum alignment which determines the structure of VEVs for the fields ζ and ξ. This section will discuss how exactly this alignment can arise. The first stage of the symmetry breaking is triggered by the ζ field acquiring a VEV radiatively. We assume that the soft mass of the ζ field gets driven negative at some scale through radiative corrections. This could be achieved if we assume the field ζ has Yukawa couplings to a massive field. Such radiative effects can trigger a VEV for ζ [18]. We have the freedom to rotate the VEV of ζ to read $\langle\zeta\rangle^T = (A, B, 0)$ without loss of generality. At this point there is nothing to say whether ξ gets a VEV or not so we assign an arbitrary structure to ξ of the form $\langle\xi\rangle^T = (D, E, F)$, where D,E and F can still be zero. The superpotential terms

$$S \sim P\zeta_i\zeta_i + T\xi_j\xi_j \tag{4}$$

[4] In this example we have no constraints on the sizes of a and d, but for $ad > 1/4$ we give N_3 a larger mass than N_1 and N_2. Consequently resonant leptogenesis could proceed via the decay of N_1 and N_2.

can be written down assuming consistent R-charge assignments (a specific example is given in later sections and in Appendix B). Along the F-flat direction $|F_P|^2 = 0$, we have $\langle \zeta^2 \rangle = 0$, which forces $A = -iB$, leading to $\langle \zeta \rangle^T = (A, Ai, 0)$. Moreover, along the F-flat direction $|F_T|^2 = 0$ we have the condition,

$$\left\langle \xi^2 \right\rangle = D^2 + E^2 + F^2 = 0 \tag{5}$$

In order to have radiative corrections generating large VEVs they must evolve along D-flat directions. The conditions for D-flatness arising from the generators

$$T_1 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad T_2 = \frac{1}{2}\begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, \quad T_3 = \frac{1}{2}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{6}$$

are of the form

$$|D_1|^2 \propto |EF^* - E^*F|^2 = 0 \tag{7}$$
$$|D_2|^2 \propto |FD^* - F^*D|^2 = 0 \tag{8}$$
$$|D_3|^2 \propto \left|2|A|^2 + i(DE^* - D^*E)\right|^2 = 0 \tag{9}$$

A solution to conditions (7) and (8) is $F = 0$. Applying this condition to (5) and rewriting the potentially complex parameters D and E as $D = D_R + iD_I$ and $E = E_R + iE_I$ we have,

$$D_R^2 - D_I^2 + E_R^2 - E_I^2 = 0 \tag{10}$$
$$D_R D_I + E_R E_I = 0 \tag{11}$$

and (9) gives

$$|A|^2 = D_I E_R - D_R E_I. \tag{12}$$

Solving conditions (10), (11) and (12) we are led to the relations

$$D_R = -E_I, \quad D_I = E_R \quad \Rightarrow \quad E = -iD. \tag{13}$$

Which means,

$$D_I = \pm\sqrt{|A|^2 - D_R^2} \tag{14}$$

where $-|A| \le D_R \le |A|$. Finally the full expression for $\langle \xi \rangle$ is

$$\langle \xi \rangle = \begin{pmatrix} D_R \pm i\sqrt{|A|^2 - D_R^2} \\ \pm\sqrt{|A|^2 - D_R^2} - iD_R \\ 0 \end{pmatrix} = \begin{pmatrix} D \\ -iD \\ 0 \end{pmatrix}. \tag{15}$$

Substituting these relations back into (7) and (8) we find $F = 0$ is a consistent solution [5].

[5] In this analysis, possible soft mass terms for the flavon fields have been neglected. If we include such terms, we will generate corrections to the vacuum alignment above which are parametrically the scale of the soft masses. We expect these corrections to be of order $\sim M_{susy}$. When we include these corrections into the VEVs of ζ and ξ we generate non-diagonal and differing (1,1) and (2,2) terms in the mass matrix of the RH neutrinos of order $\sim M_{susy}^2/M_f$ at most.

3 A Toy Model

The aim of this section is to show that the SO(3) flavour symmetry can be used in a model that successfully describes all fermionic sectors including the generation of neutrino masses. We do this using, along side the SO(3) flavour symmetry, an adaptation of the model described in Ref.[18]. In this paper all the MSSM fields including the RH neutrino field are triplets under an SU(3) flavour symmetry. However in our adaptation the RH neutrino fields are now singlets under the SU(3) flavour symmetry and a triplet under the new SO(3) flavour symmetry. The other MSSM fields are singlets under the SO(3) flavour symmetry. Summarising, the flavour symmetry assignments we have for the SO(3) symmetry,

$$(Q, L, U^c, D^c, E^c) \sim 1, \qquad N_i \sim 3 \tag{16}$$

and for the SU(3) symmetry

$$(Q_\alpha, L_\alpha) \sim 3, \qquad (U^c_\alpha, D^c_\alpha, E^c_\alpha) \sim \bar{3}, \qquad N \sim 1 \tag{17}$$

where $\alpha = 1, 2, 3$ (and all other Greek indices) are SU(3) labels. Moreover, all Higgs fields responsible for SU(3) symmetry breaking as well as any other fields used to achieve the desired vacuum alignment are singlets under the new SO(3) flavour symmetry. A summary of all the assignments is given in Appendix A. We use the mechanisms presented in Ref.[18] for all sectors apart from the neutrino sector which we present here.

3.1 Neutrino masses from Susy breaking

We need to generate neutrino masses and we do this in a similar way to Ref.[16]. As emphasized by the authors of Ref.[15], we can apply the Giudice-Masiero mechanism [19] to the neutrino sector, i.e SM-singlet operators, such as the RH neutrino mass $M_R NN$, or the neutrino Yukawa coupling $\lambda L N H_u$, might only appear to be renormalizable superpotential terms, but in fact may arise from $1/M$-suppressed terms involving the fundamental supersymmetry breaking scale $m_I \sim \sqrt{M_{3/2} M_{pl}}$, where M_{pl} and $M_{3/2}$ are the reduced Planck mass and gravitino mass respectively.

Specifically, consider the usual MSSM Lagrangian to be supplemented by Standard-Model-singlet chiral superfields which arise from the hidden sector. In general, the fields which communicates supersymmetry breaking to the neutrinos can either be flavour singlets or flavour non-singlets. Here we assume that all such fields are singlets under all flavour symmetries.

Ignoring flavour and consequently suppressing all indices for the moment, the scales of the various terms we wish to study are set by the hidden sector fields acquiring VEVs. In the superpotential we have

$$\mathcal{L}_N^W = \int d^2\theta \left(g \frac{T}{M} L N H_u \right), \tag{18}$$

while the set of terms involving the RH neutrino fields in the Kahler potential are

$$\mathcal{L}_N^K = \int d^4\theta \left(h\frac{T^\dagger}{M}NN + \tilde{h}\frac{T^\dagger T}{M^2}N^\dagger N + h_B\frac{T^\dagger T T^\dagger}{M^3}NN + \ldots \right). \tag{19}$$

Here T is a susy breaking hidden sector field and the ellipses in (19) stand for terms higher order in the $1/M$-expansion. It is simple to check the additional terms will lead to trivial or sub-dominant contributions not relevant for our discussion. All dimensionless couplings g, h, etc, are taken to be $O(1)$.

Let us now suppose that after supersymmetry is broken in the hidden sector at the scale m_I, the field T acquires the following F- and A-component VEVs,

$$\begin{aligned} \langle T \rangle_F &= F_t = f_t m_I^2 \\ \langle T \rangle_A &= A_t = a_t m_I. \end{aligned} \tag{20}$$

Here f_t and a_t are $O(1)$. Substituting these VEVs into Eq. (18) and (19) shows that after susy breaking we produce; (1) the scale for neutrino Yukawa as $\sim 10^{-7} - 10^{-8}$, (2) RH neutrino mass scale at a TeV, (3) a trilinear scaler A-term at a TeV, (4) RH sneutrino lepton-number violating B-term with magnitude $B^2 \sim (\text{few} \times 100\,\text{MeV})^2$. We produce two sources of neutrino masses, a tree level (see-saw) contribution as well as a dominant 1-loop contribution, ([15], [16]). In the next section we outline how one could combine the susy breaking model described above with the flavour symmetries, SO(3) and SU(3) to give neutrino masses and mixings compatible with current experimental bounds.

3.1.1 RH Neutrino Mass Matrix

In the susy breaking model described above the RH neutrino mass terms arise from non-renormalisable Kahler potential operators. In order to produce degenerate RH neutrinos this way consider,

$$K \sim \frac{T^\dagger}{M_{pl}} \left(h_4 N_i N_i + \frac{1}{M_f^2} h_5 N_i \zeta_i N_j \zeta_j^* + \frac{1}{M_f^2} h_6 N_i \xi_i N_j \xi_j^* \right) \tag{21}$$

$$+ \frac{T^\dagger}{M_{pl}} \left(h_7 \frac{1}{M_f^4} \epsilon_{ijk} N_i \zeta_j \xi_k \epsilon_{lmn} N_l \zeta_m^* \xi_n^* + \ldots \right) \tag{22}$$

where the ellipses represent further terms that do not contribute to non-diagonal terms or give differing (1,1), (2,2) entries. We assume the R-charge assignments in Table 1 of Appendix A. Allowing the flavon fields to gain their appropriate VEVs, the RH neutrino mass matrix takes the following form,

$$M_N \sim \begin{pmatrix} h_4 + h_5 |a|^2 + h_6 |d|^2 & 0 & 0 \\ 0 & h_4 + h_5 |a|^2 + h_6 |d|^2 & 0 \\ 0 & 0 & h_4 + h_7' |a|^2 |d|^2 \end{pmatrix}, \tag{23}$$

generating two exactly degenerate RH neutrinos. h_7' represents the fact that there are numerous terms of the same order as the term in (22) contributing to the mass[6] of N_3.

[6]In order to be consistent with neutrino masses and mixings, we take parameter values $a = d = 0.4$. Even with these values the mass of N_3 is larger than that of N_1 and N_2 due to these additional terms.

3.1.2 Trilinear Scaler A-term

A very important term which contributes to the 1-loop neutrino masses is the trilinear scalar A-term. The structure of this term comes from the following leading order superpotential operators,

$$S_A \sim \frac{T}{M_{pl}}\left(g_1\frac{1}{M_f^4}\epsilon_{ijk}N_i\zeta_j\xi_k\frac{1}{M_3^7}L_\alpha\phi_3^\alpha(\overline{\phi}_3\phi_3)^3(\zeta\xi)\right) \tag{24}$$

$$+\frac{T}{M_{pl}}\left(g_2\frac{1}{M_f^2}\epsilon_{ijk}N_i\zeta_j\xi_k\frac{1}{MM_3^8}L_\alpha\phi_{23}^\alpha(\overline{\phi}_3\phi_3)^4\right) \tag{25}$$

$$+\frac{T}{M_{pl}}\left(g_3\frac{1}{M_f^4}\epsilon_{ijk}N_i\zeta_j\xi_k\frac{1}{MM_3}\epsilon^{\alpha\beta\gamma}L_\alpha\overline{\phi}_{23,\beta}\overline{\phi}_{3,\gamma}(\zeta\xi)+\dots\right) \tag{26}$$

Giving the structure,

$$A_\nu \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ g_3 4a^2d^2\epsilon i & g_2 2ad\epsilon i & g_1 4a^2d^2 i \end{pmatrix} \tag{27}$$

where we have written $\epsilon = b/M$ and ϵ, a and d are expansion parameters. Here we assume that the ϵ parameter can be different to the expansion parameter for the up quark sector. The neutrino sector is generated via non-renormalisable susy breaking operators, with the RH neutrino transforming as a singlet under the SU(3) flavour symmetry in contrast to Ref. [18] where the expansion parameters are identical for the two sectors.

3.1.3 Neutrino Yukawa term

In order to generate neutrino masses and mixings it is necessary to add two hidden sector superfields, Z_1 and Z_2, with properties and charge assignments as listed in Table 1 of Appendix A. Specifically we assume the Z fields gain A-component VEVs, $\langle Z\rangle_A = A_z = a_z m_I$, with zero (or tiny) F-component VEVs.

The Yukawa flavour structure has a contribution from the new fields, Z_1 and Z_2 in addition to a contribution from the field T. The contribution from the field T has exactly the same structure as the trilinear scaler A-term except for the Yukawa the A-component VEV of T is used. Leading order contributions from fields Z_1 and Z_2 are,

$$S_{Yuk} \sim \left(\frac{Z_1}{M_{pl}}g_4\frac{1}{M_f}N_i\zeta_i + \frac{Z_2}{M_{pl}}g_7\frac{1}{M_f}N_i\xi_i\right)\frac{1}{MM_3^{10}}L_\alpha\phi_{23}^\alpha(\overline{\phi}_3\phi_3)^5 \tag{28}$$

$$+\left(\frac{Z_1}{M_{pl}}g_5\frac{1}{M_f}N_i\zeta_i + \frac{Z_2}{M_{pl}}g_8\frac{1}{M_f}N_i\xi_i\right)\frac{1}{M_3^9M_f^2}L_\alpha\phi_3^\alpha(\overline{\phi}_3\phi_3)^4(\zeta\xi) \tag{29}$$

$$+\left(\frac{Z_1}{M_{pl}}g_6\frac{1}{M_f}N_i\zeta_i + \frac{Z_2}{M_{pl}}g_9\frac{1}{M_f}N_i\xi_i\right)\frac{1}{MM_3^3M_f^2}\epsilon^{\alpha\beta\gamma}L_\alpha\overline{\phi}_{23,\beta}\overline{\phi}_{3,\gamma}(\overline{\phi}_3\phi_3)(\zeta\xi). \tag{30}$$

Giving the leading order Yukawa structure,

$$\begin{pmatrix} (a_{Z1}g_6a + a_{Z2}g_9d)2ad\epsilon & (a_{Z1}g_4a + a_{Z2}g_7d)\epsilon & (a_{Z1}g_6a + a_{Z2}g_9d)2ad \\ (a_{Z1}g_6a - a_{Z2}g_9d)2iad\epsilon & (a_{Z1}g_4a - a_{Z2}g_7d)\epsilon i & (a_{Z1}g_6a - a_{Z2}g_9d)2iad \\ g_3a_T4a^2d^2\epsilon i & g_2a_T2ad\epsilon i & (g_2\epsilon + g_12ad)a_T2adi \end{pmatrix}. \tag{31}$$

3.1.4 Other Terms of Note

The lepton-number violating B-term is crucial to the formation of the 1-loop contribution to the light neutrino masses. The structure of the B-term assuming a and d are real for simplicity, is

$$\begin{pmatrix} (h_4 + h_5 a^2 + h_6 d^2) a_t & 0 & h_{16} i a^2 d (a_{z1} + h'_{16} a_{z2}) \\ 0 & (h_4 + h_5 a^2 + h_6 d^2) a_t & h_{16} a^2 d (a_{z1} - h'_{16} a_{z2}) \\ h_{16} i a^2 d (a_{z1} + h'_{16} a_{z2}) & h_{16} a^2 d (a_{z1} - h'_{16} a_{z2}) & (h_4 + h_8) a_t \end{pmatrix} \tag{32}$$

which we generate from operators of the form of the 3rd term in equation (19) and similar operators with one of the $T^\dagger$s being replaced by a $Z^\dagger$.

We can also generate small corrections to the RH neutrino mass matrix using the same form of operator. This is achieved when $T^\dagger$ gets an F-component VEV and two other hidden sector fields get A-component VEVs. (The other two hidden fields could be $T^\dagger T$,$Z^\dagger Z$,$T^\dagger Z$ or $Z^\dagger T$.) The resulting structure of this splitting term, ΔM_N, in the limit where $a \sim d$,

$$\begin{pmatrix} (h_4 + h_5 a^2 + h_6 d^2) a_t & 0 & i a^3 a_3^2 (a_{z1} h_{18} + a_{z2} h_{18}) \\ 0 & (h_4 + h_5 a^2 + h_6 d^2) a_t & a^3 a_3^2 (a_{z1} h_{18} - a_{z2} h_{18}) \\ i a^3 a_3^2 (a_{z1} h_{18} + a_{z2} h_{18}) & a^3 a_3^2 (a_{z1} h_{18} - a_{z2} h_{18}) & (h_4 + h_8) a_t \end{pmatrix} \tag{33}$$

with a scale of $\sim 10^{-13}$ GeV and where numerical factors have been ignored. These splittings actually play no significant role in splitting of the RH neutrinos as they enter into the matrix as mixings between the 1st and 3rd and 2nd and 3rd generations.

3.2 Neutrino Masses and Mixings

As is described in Ref.[16] neutrino masses can be generated from two different sources. The dominant piece is that produced by a 1-loop contribution. The flavour structure of this contribution in the limit that there is no mixing in the sneutrino sector is,

$$m_\nu^{\rm loop} \sim A^T B^* A \tag{34}$$

Substituting in the forms for A and B from Eqs. (27) and (32) respectively we get the structure,

$$m_\nu^{\rm loop} \sim a^2 d^2 \begin{pmatrix} ad\epsilon^2 & ad\epsilon^2 & ad\epsilon^2 + a^2 d^2 \epsilon \\ ad\epsilon^2 & \epsilon^2 & \epsilon^2 + ad \\ ad\epsilon^2 + a^2 d^2 \epsilon & \epsilon^2 + ad & \epsilon^2 + a^2 d^2 + ad\epsilon \end{pmatrix} \tag{35}$$

where numerical factors and various h and g coefficients have been suppressed for simplicity. The form of this neutrino mass can be identified with the structure for a normal hierarchy of neutrino masses. On its own it can successfully generate the atmospheric neutrino mass data. However in its current form it is rank 1. We now need the second source of neutrino masses which comes from the tree level "see-saw" contribution. This has the form,

$$(m_\nu^{\rm tree})_{ij} = -v^2 \sin^2 \beta \lambda_{ik}^T M_N^{-1} \lambda_{kj}. \tag{36}$$

This tree level contribution provides a useful perturbation to the 1-loop structure and provides the solar neutrino mass scale in this case. Combining these two sources of neutrino mass we can produce neutrino masses with a normal hierarchy. Assuming reasonable values for the various g and h coefficients (which can be complex) and with $a \sim b \sim 0.4$, $\epsilon \sim 0.20$ it is possible to achieve mass splittings compatible with measured values (an appropriate diagonalisation procedure for a hierarchical mass matrix is outlined in Ref.[20]). Due to the large value of the (2,2) component of $m_\nu^{\rm loop}$ compared to the value of the (1,1) component, we do not naturally produce large values for θ_{12}. Consequently we need to moderately fine tune some of the g and h coefficients in order to produce consistent mixing angles. Assuming the mixing angles from the charged lepton sector are small, the resulting MNS mixing angles produced from the neutrino sector can accommodate the oscillation data. The analysis given in Ref. [18] suggests small corrections from the charged lepton sector are possible within the SU(3) flavour scenario.

3.3 TeV scale Leptogenesis from Susy breaking

In this model we have large tri-linear scaler A-terms and consequently the RH sneutrinos will be in deep thermal equilibrium at a scale $\sim M_{\tilde{N}_i}$. Therefore the decay of the sneutrinos cannot lead to the creation of a large asymmetry. The RH neutrinos on the other hand are not in the thermal equilibrium due to the tiny effective Yukawa couplings. In addition, the tree level vertex diagram for the decay of the RH neutrinos is negligible compared to the self energy diagram shown in Fig.1 of Ref.[9], which is responsible for the asymmetry. Although the diagram is suppressed by the Yukawa couplings it is enhanced by a resonance effect when the mass splittings are naturally tiny as they are for two of the RH neutrinos in the SO(3) model described in this letter. The form of the total asymmetry is [5, 7, 8],

$$\varepsilon_{tot} = \sum_i \varepsilon_i = \sum_i \left(- \sum_{j \neq i} \frac{M_i}{M_j} \frac{\Gamma_j}{M_j} I_{ij} S_{ij} \right) , \tag{37}$$

where

$$I_{ij} = \frac{\mathrm{Im}\,[(\lambda^{(1)}\lambda^{(1)\dagger})^2_{ij}]}{|\lambda^{(1)}\lambda^{(1)\dagger}|_{ii}|\lambda^{(1)}\lambda^{(1)\dagger}|_{jj}} , \qquad S_{ij} = \frac{M_j^2 \Delta M_{ij}^2}{(\Delta M_{ij}^2)^2 + M_i^2 \Gamma_j^2} , \qquad \Gamma_j = \frac{|\lambda^{(1)}\lambda^{(1)\dagger}|_{jj}}{8\pi} M_j . \tag{38}$$

Where $\lambda^{(1)} = U_N \lambda$ are the 1-loop corrected Yukawa couplings [7] with U_N the unitary matrix that diagonalises the full contribution to the RH neutrino mass matrix,

$$M_N^R = M_N + \beta(M_N \lambda\lambda^\dagger + \lambda^* \lambda^T M_N) + \gamma \Delta M_N \tag{39}$$

where[8]

$$\beta \sim \frac{m_{3/2}}{h M_P} \left(\frac{g^2}{16\pi^2} \log \frac{M_P}{M_N} \right) \sim 10^{-15} \tag{40}$$

[7]Resummations of the Yukawa couplings have not been performed for simplicity, an example of such a procedure in the context of resonant leptogenesis is given in Ref.[14].

[8]Note that the definitions of β and γ are modified compared to those given in Ref.[16].

and

$$\gamma = \frac{m_{3/2}^2}{M_P} \sim 10^{-12}. \tag{41}$$

Diagonalising M_N^R gives a mass splitting in the first two generations that is the same parametric size as the width for these states. This produces a resonance in the propagator of the virtual RH neutrino in the self energy diagram for N_1 and N_2. This does not happen when N_3 is present due to the much larger mass splitting between N_3 and the other RH neutrino generations. Consequently, we only get two pieces contributing significantly to ε_{tot},

$$\varepsilon_{tot} \simeq \frac{M_1}{M_2}\frac{\Gamma_2}{M_2} I_{12} S_{12} + \frac{M_2}{M_1}\frac{\Gamma_1}{M_1} I_{21} S_{21} \tag{42}$$

rearranging to give

$$\varepsilon_{tot} \simeq \frac{M_1 M_2 I_{12}}{8\pi} \Delta M_{12}^2 \left[\frac{|\lambda^{(1)}\lambda^{\dagger(1)}|_{22}}{(\Delta M_{12}^2)^2 + M_1^2 \Gamma_2^2} + \frac{|\lambda^{(1)}\lambda^{\dagger(1)}|_{11}}{(\Delta M_{12}^2)^2 + M_2^2 \Gamma_1^2} \right]. \tag{43}$$

Using the same coefficients that were used to construct the neutrino sector we find that we are actually a little bit off resonance, such that $(\Delta M_{12}^2)^2 > M^2\Gamma^2$. The actual size of the mass splitting is of the order $\sim 10^{-8}\,\mathrm{GeV}^2$. This is a little bigger than we might expect from the parametric sizes of the non-diagonal RH neutrino contributions in eq. (39). The large size is due to the large mixing angle generated in the 1st two generations as a result of the high degree of degeneracy in the masses at tree level. We also have that $|\lambda^{(1)}\lambda^{\dagger(1)}|_{22} \sim |\lambda^{(1)}\lambda^{\dagger(1)}|_{11}$. Applying this we have,

$$\varepsilon_{tot} \simeq \frac{M_1 M_2 I_{12}}{4\pi} \frac{|\lambda^{(1)}\lambda^{\dagger(1)}|_{22}}{\Delta M_{12}^2}. \tag{44}$$

Inserting, $\Delta M_{12}^2 \sim 10^{-8}\,\mathrm{GeV}^2$, $|\lambda^{(1)}\lambda^{\dagger(1)}|_{22} \sim 10^{-14}$ and $M_i \sim 10^2\,\mathrm{GeV}$ we have

$$\varepsilon_{tot} \sim I_{12} 10^{-2}. \tag{45}$$

The off-diagonal parts of $\lambda^{(1)}\lambda^{\dagger(1)}$, with these parameters, are small compared to the diagonal parts due to non-trivial cancellations, consequently, I_{12} comes out to be of order 10^{-5}, giving,

$$\varepsilon_{tot} \sim 10^{-7}. \tag{46}$$

Due to the sizes of the Yukawa couplings the decay widths of the RH neutrinos are less than the Hubble constant and therefore will not induce any wash-out effects via decays or scatterings. The large A-terms do not contribute to any wash-out effects as they need to be accompanied by a Yukawa interaction or a lepton number violating B-term interaction (which is also small) in order to break lepton number. Thus with $g^* \sim 100$, n_L/s can be of order $\varepsilon_{tot}/100 \sim 10^{-9}$ which is at the correct order to give the CMBR-determined experimental value, $n_B/n_\gamma = 6.1^{+0.3}_{-0.2} \cdot 10^{-10}$ [21].

4 Conclusions

In the context of supersymmetric theories, a weakly broken gauged SO(3) flavour symmetry was used to produce two highly degenerate RH neutrinos. It was shown that this

SO(3) flavour symmetry is compatible with all fermion masses and mixings if it is supplemented with a further SU(3) flavour symmetry. A specific susy breaking model was then used to generate the light neutrino masses as well as a natural model of TeV scale resonant leptogenesis. It must be noted that this SO(3) flavour symmetry and its associated flavon field alignments can be used independently of the susy breaking model used to produce the neutrino masses in this letter. An application of this was given in section 2 where degenerate RH neutrinos were generated in the context of an effective superpotential.

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Appendix A

Below we list the assignments of all the fields in the theory.

Field	R-Charge	R-Parity	Z_2	SU(3)	SO(3)	VEV
ζ^T	$-1/5$	$+$	$-$	1	3	$(A, iA, 0)$
ξ^T	$-4/5$	$+$	$+$	1	3	$(D, -iD, 0)$
ϕ_3^T	1	$+$	$+$	$\bar{3}$	1	$(0,0,a_3)$
ϕ_{23}^T	1	$+$	$-$	$\bar{3}$	1	$(0,b,b)$
$\overline{\phi}_2$	0	$+$	$+$	3	1	$(0,a_2,0)$
$\overline{\phi}_3$	-2	$+$	$+$	3	1	$(0,0,a_3)$
$\overline{\phi}_{23}$	0	$+$	$+$	3	1	$(0,b,-b)$
T	4/3	$+$	$+$	1	1	(Fcpt,Acpt)=$(m_I^2 f_t, m_I a_t)$
Z_1	23/15	$+$	$+$	1	1	(Fcpt,Acpt)=$(0, m_I a_{z1})$
Z_2	32/15	$+$	$-$	1	1	(Fcpt,Acpt)=$(0, m_I a_{z2})$
T	12/5	$+$	$+$	1	1	-
P	18/5	$+$	$+$	1	1	-
N	2/3	$-$	$+$	1	3	-
L	4	$-$	$+$	3	1	-
Q	0	$-$	$+$	3	1	-
U^c	0	$-$	$+$	3	1	-
D^c	0	$-$	$+$	3	1	-
E^c	-4	$-$	$+$	3	1	-
H_u	0	$+$	$+$	1	1	v_2
H_d	0	$+$	$+$	1	1	v_1

Table 1: Table of field assignments

Appendix B

Due to the R-charge assignments of the SO(3) flavon fields there are terms that can be written down in addition to those in equation (4). The additional terms are,

$$PT\frac{(\zeta_i\xi_i)^4}{M^7} + PT\frac{(\zeta_i\xi_i)^2(\zeta_j\zeta_j)(\xi_k\xi_k)}{M^7} + PT\frac{(\zeta_j\zeta_j)^2(\xi_k\xi_k)^2}{M^7} + T\frac{(\zeta_j\zeta_j)^4}{M^6}. \tag{B-1}$$

Along the F-flat direction $|F_P|^2 = 0$, we now have,

$$\langle\zeta_i\zeta_i\rangle + \frac{\langle T(\zeta_i\xi_i)^4\rangle}{M^7} + \frac{\langle T(\zeta_i\xi_i)^2(\zeta_j\zeta_j)(\xi_k\xi_k)\rangle}{M^7} + \frac{\langle T(\zeta_j\zeta_j)^2(\xi_k\xi_k)^2\rangle}{M^7} = 0 \tag{B-2}$$

leading to $\langle\zeta^2\rangle = 0$ and $\langle T\rangle = 0$. Along the F-flat direction $|F_T|^2 = 0$ applying $\langle\zeta^2\rangle = 0$ we have the condition,

$$\langle\xi_i\xi_i\rangle + \frac{\langle P(\zeta_i\xi_i)^4\rangle}{M^7} = 0 \tag{B-3}$$

leading to $\langle\xi^2\rangle = 0$ and $\langle P\rangle = 0$, which are the conditions we require for the correct vacuum alignment.

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